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THE MINIMAL MODEL PROGRAM IN CHARACTERISTIC P

EDITED BY

This workshop, sponsored by AIM and the NSF, was devoted to the minimal model program in characteristic p.

Despite recent progress in characteristic zero in all dimensions relatively little is known about the birational geometry of varieties in characteristic p, even for threefolds. Kawamata-Viehweg vanishing is one of the central results in characteristic zero but unfortunately it is known that Kodaira vanishing fails even for surfaces in characteristic p.

The singularities which appear in the minimal model program are adapted to the use of Kawamata-Viehweg vanishing. In characteristic p there are some closely related singularities which arise naturally when considering the action of Frobenius. One aim of the workshop was to understand how the two types of singularities compare.

Using ideas and techniques from characteristic zero coupled with some recent progress on alternatives to Kawamata-Viehweg vanishing in characteristic p, which use the action of Frobenius, one of the aims of the workshop was to attack problems in the birational geometry of threefolds and possibly even higher dimensions in characteristic p.

The main topics of the workshop were:

1. Vanishing theorems in finite characteristic.
2. The cone and base point free theorem in characteristic p.
3. Existence of three fold flips in characteristic p.
4. Semi-stable reduction for surfaces in characteristic p.
5. Boundedness of birational maps for threefolds.
6. The behavior of nef divisors modulo reduction to characteristic p.

1. Fundamental MMP theorems

Analogs of the fundamentals theorems of the MMP in characteristic 0 whose positive characteristic forms remain open.

Basepoint free theorem

Problem .1. *Suppose that $k = \bar{k}$, char $k = p > 0$. Let X/k be a terminal threefold, and A an ample $\mathbb{Q}$-divisor on X. If $K_X + A$ is nef, must it be semiample?*

Termination in dimension 3

Problem .2. *Do klt flips terminate in dimension 3?*

Connectedness

Problem .3. *Is there a positive-characteristic analog of the connectedness lemma?*

2. Singularities in char p

Questions about singularities of the MMP in characteristic p.

Terminal and Cohen-Macaulay

Problem .1. *Are (log) terminal singularities Cohen-Macaulay? Rational?*

Normality of plt pairs

Problem .2. *Suppose that $(X, D = S + B)$ is a plt pair. Must S be normal?*

3. Geometry of F-singularities

Questions dealing with the properties of F-singularities

Grauert-Riemenschneider for F-regular varieties

Problem .1. *Is there Grauert-Riemenschneider vanishing for F-regular varieties which admit a resolution?*

Global F-regularity and rational chain connectedness

Problem .2. *Does globally F-regular imply rationally chain connected?*

Global F-regularity and Fano type

Problem .3. *If X is of globally F-regular type, does it follow that it is log Fano (in the sense that there exists a boundary divisor Δ with (X, Δ) klt and $-(K_X + \Delta)$ ample)? Even weaker, does it follow that $-K_X$ is big?*

Construction of F-pure centers

Problem .4. *Suppose that (X, Δ) is a strongly F-regular pair. Can we find a boundary $D \geq 0$ such that $(X, \Delta + D)$ is F-pure, with $p \nmid index(K_X + \Delta + D)$?*

4. Sections and section rings

Questions about the existence of sections, and properties of various section rings.

Fano and MDS

Problem .1. *Is every Fano variety a Mori Dream Space?*

Invariance of plurigenera

Problem .2. *What is the status of invariance of plurigenera?*

General type in families

Problem .4. *Suppose that $\mathcal{X} \to \Delta$ is a family, and a special fiber is of general type. Does it follow that a general fiber is of general type?*

Problem .3. *What if we look at dim $S^0(mK_X + A)$ instead?*

Embedding by $S^0(mK_X)$

Problem .5. *Suppose that X is of general type. Is there an effective constant $m = m(n)$ such that $S^0(mK_X)$ (the canonical linear system of Schwede) defines a birational map? What about the usual linear system $|mK_X|$?*

Effective Fujita vanishing

Problem .6. *Is there an effective Fujita vanishing result in flat families? Suppose that $f : \mathcal{X} \to S$ is flat, $\mathcal{F}$ is a coherent sheaf on $\mathcal{X}$, and $\mathcal{L}$ is f-ample. Does there exist a constant $m_0 = m_0(\mathcal{X}, \mathcal{F}, \mathcal{L})$ such that $H^i(X_s, (\mathcal{F} \otimes \mathcal{L}^m)|_{X_s}) = 0$ if s is any point of S, $m \geq m_0$ and $\mathcal{M}$ is any f-nef line bundle on X?*

Sections of nef $K_X + A$

Problem .7. *Suppose that A is Cartier and ample, and $K_X + A$ is nef. Must $H^0(X, K_X + A)$ be nonzero?*

F-regularity and finite generation

Problem .8. *Suppose that X is an F-regular variety (maybe not $\mathbb{Q}$-Gorenstein). If D is a Weil divisor, must $\bigoplus_{m\geq 0} \mathcal{O}_X(mD)$ be finitely generated?*

5. OTHER QUESTIONS

Analogs of some other questions from characteristic 0.

Pulling back forms to a resolution

Problem .1. *Suppose that X is log canonical, and that there exists a log resolution $f : \tilde{X} \to X$ which is an isomorphism over the smooth locus. Let ω be an $(n-1)$-form on X. Does $f^*\omega|_{X_{smooth}}$ extend to an $(n-1)$-form on $\tilde{X}$ with log poles along the exceptional locus?*

Universal lower bounds for Seshadri constants

Problem .2. *Suppose that X is a smooth variety over an uncountable field k of positive characteristic. Does there exist a constant $c = c(n)$ such that $\epsilon(L, x) \geq c$ for any ample divisor L and very general point x of X?*

Nefness under mod p reduction

Problem .3. *Suppose that X is a variety over k, with char $k = 0$, and L is a nef divisor on X. Must L_p be nef on X_p for infinitely many p? What about in the case $L = K_X$? What if "nef" is replaced by "semiample"?*

Singularities and point-counting

Problem .4. *Suppose that X is a variety over a finite field. Is there any relation between the singularities of X and the number of points over $\mathbb{F}_q$? For example, suppose that X is Fano and F-regular.*

REFERENCES