[2007/11/11]

DEFORMATION THEORY AND THE BRAUER GROUP

EDITED BY DANIEL KRASHEN AND MAX LIEBLICH

This problem list begin during discussions at the AIM workshop Deformation theory, patching, quadratic forms, and the Brauer group in January, 2011.

1. THE u-INVARIANT PROBLEM

The problems in this section are related to the u-invariant problem on the maximal dimension of an anisotropic quadratic form.

Problem .1. *If $u(F) < \infty$, does there exist $B \in \mathbb{N}$ such that $u(F') < \infty$ for all F'/F finite? In particular, is $B = 2u(F)$ sufficient?*

Problem .6. *When is τ_F finite?*

Problem .2. *Find interesting classes $\mathcal{F}$ of fields such that for $F \in \mathcal{F}$,*

$$u(F) < \infty \implies u(F(t)) < \infty.$$

Problem .7. *When $F = \mathbb{R}$, we have $u(F) = \infty$ but $\tau_F < \infty$. Is there an example where this holds for F not formally real?*

Define

$$u_k(F) = \max\{\dim(q) \mid q \in I^k\}.$$

Then $u_0 = u$.

Problem .3. *Can we compute $u_k(F)$? How about $u_3(F)$?*

Problem .8. *Find reasonable classes $\mathcal{F}$ of fields such that $u(F)$ is a power of 2 for all $F \in \mathcal{F}$. For these fields, assume k is the period-index bound for $l = 2$. Does this imply $u(F) = 2^{k+1}$?*

Remark. I think I've seen this somewhere before

Let

$$\nu(F) = \max\{n \mid I^n \neq 0\}.$$

Problem .4. *If $v(F) < \infty$, is it true that $v(F') < v(F) + 1$ for all F'/F finite?*

Remark. $2^{v(F)} < u(F)$

Remark. This is related to the computation of Galois groups of quadratically closed fields.

For a quadratic form q over F, we define

$$\text{splitting degree} = \min\{[L : F] \ : \ q_L \text{ is a direct sum of hyperbolics}\}.$$

The *torsion index* of F, denoted τ_F is the maximum splitting degree, taken over all even dimensional q.

Problem .5. *Is $\tau_F < 2^{(\frac{u(F)}{2}-1)}$?*

2. The Brauer group

Problem .1. *Fix $d \geq 3$. Assume that we know:*
$\text{ind}(\alpha) | \text{Per}(\alpha)^{d-1}$*, for all $\alpha \in \text{Br}(X)$, for all smooth, projective X over $\overline{\mathbb{F}_p}$. (Here $d = \dim(X)$.)*
Can we use this, by a boundedness argument, to show the same thing for $\alpha \in \text{Br}(Y)$, where Y smooth, projective, $\dim(Y) = d$ *over $\mathbb{C}$?*

Problem .2. *Is every Brauer class over $\mathbb{C}(s,t)$ cyclic?*

Remark. This type of question does not reduce to the prime case.

A variant of Problem 2.2:

Problem .3. *Higher cohomology $H^d(F, \boldsymbol{\mu}_n^{\otimes d})$, where $d = \dim(F)$.*

Problem .4. *When is* $\text{Br}(X) = \text{Br'}(X)$*?*

Remark. If X is quasi-proj, then yes.

3. Testing section

You can use this section for testing, or we can just delete it.
Rename it to something useful if you need another section.

Name of the problem block

Intro to problem block

Conjecture 3.1. *This is the problem statement.*

Remark. This is a remark.

Test introduction

Problem 3.2. *the statement of a throw-away problem*

REFERENCES