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COMPLEXITY OF CR MAPPINGS

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problem list

1. MAPPINGS BETWEEN BALLS

mapping problem 1

Problem 1.1. *Let $n \geq 2$.*
Given a proper rational map $f : \mathbb{B}^n \to \mathbb{B}^N$, find the smallest number $k(n,N)$ such that f is determined by its k-jet at the origin.
Let Ω, Ω' in $\mathbb{C}^n$ and $\mathbb{C}^N$ respectively be strongly pseudoconvex domains. If $f : \Omega \to \Omega'$ is a proper holomorphic map which extends smoothly to $\partial\Omega$, does there exist a k such that f is determined by its k-jet at a point?

Problem 1.2. *Assume $M \subseteq \mathbb{C}^n$ is a real-analytic generic submanifold, $f = \frac{p}{q} : \mathbb{C}^n \to \mathbb{C}^N$ is the germ of a meromorphic map at $z \in M$, and f is holomorphic in a one-sided wedge W attached to M near z. Assume that $\|f(w)\|^2$ increases to 1 as $w \to M$ in W. Does f extend to a full neighborhood of z in $\mathbb{C}^n$?*

Remark. the codimension 1 case is known (Chiappari, '91)

Problem 1.3. *Does there exist a universal constant t such that a proper holomorphic map between $\mathbb{B}^n$ and $\mathbb{B}^N$ (with $1 < n < N$) which is C^t up to the boundary is actually a rational map?*

Problem 1.4. *Assume $\Gamma \subseteq Aut(\mathbb{B}^N)$ is a discrete subgroup and $f : \mathbb{B}^n \to \mathbb{B}^N$ is a proper holomorphic embedding such that $\Gamma(f(\mathbb{B}^n)) \subseteq f(\mathbb{B}^n)$ and $f(\mathbb{B}^n)/\Gamma$ is compact. Is f linear?*

Problem 1.5. *Let $f : \mathbb{B}^n \to \mathbb{B}^N$ be a proper rational map. Is f homotopic to a polynomial map p of the same degree? That is, $\forall t \in [0,1]$, each $f_t : \mathbb{B}^n \to \mathbb{B}^N$ is proper with $f_0 = f$ and $f_1 = p$.*

REFERENCES